THE SKYRME MODEL REVISITED: AN EFFECTIVE THEORY APPROACH AND APPLICATION TO THE PENTAQUARKS*

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The Skyrme model is reconsidered from an effective theory point of view. Starting with the most general Lagrangian up to including terms of order p^4, N_c and δm^2 ($\delta m \equiv m_s - m$), we obtain new interactions, which have never been discussed in the literature. We obtain the parameter set best fitted to the low-lying baryon masses by taking into account the representation mixing up to **27**. A prediction for the mainly anti-decuplet excited nucleon N' and Σ' is given.

1. Introduction and Summary

The narrowness of the newly discovered exotic baryonic resonance Θ^+ [1,2,3,4] has been a mystery. The direct experimental upper bound is $\Gamma_\Theta < 9$ MeV, while some re-examinations [5,6,7,8] of older data suggest $\Gamma_\Theta < 1$ MeV. At this moment, it is not very clear what makes the width so narrow.

Interestingly, the mass and its narrow width had been predicted by Diakonov, Petrov, and Polyakov[9]. Compare their predicted values, $M_\Theta =$ 1530 MeV and $\Gamma = 15$ MeV (or 30 MeV[10,11,12]), with the experimental ones[13], $M_\Theta = 1539.2 \pm 1.6$ MeV and $\Gamma = 0.9 \pm 0.3$ MeV. It is astonishing! What allows the authors to predict these numbers? It deserves a serious look.

Their predictions are based on the "chiral quark-soliton model[14]," (χQSM) which may be regarded as a version of the Skyrme model[15] with

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specific symmetry breaking interactions[a],

$$\alpha D^{(8)}_{88} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i} J_i, \tag{1}$$

where $D^{(8)}_{\alpha\beta}(A) = \frac{1}{2}\mathrm{Tr}\left(A^\dagger \lambda_\alpha A \lambda_\beta\right)$, Y is the hypercharge operator, and J_i is the spin operator. Is this a general form of the symmetry breaking? Is it possible to justify it without following their long way, just by relying on a more general argument? What is the most general Skyrme model? Is it possible to have a "model-independent" Skyrme model? This is our basic motivation.

A long time ago, Witten[16] showed that a soliton picture of baryons emerges in the large-N_c limit[17] of QCD. If the large-N_c QCD has a close resemblance to the real QCD, we may consider an effective theory (not just a model) of baryons based on the soliton picture, which may be called as the "Skyrme-Witten large-N_c effective theory." The question is in which theory the soliton appears.

A natural candidate seems the chiral perturbation theory (χPT), because it represents a low-energy QCD at least in the meson sector. Note that it is different from the conventional Skyrme model, which contains only a few interactions. We have now an infinite number of terms. We have to systematically treat these infinitely many interactions. Because we are interested in the low-energy region, we only keep the terms up to including $\mathcal{O}(p^4)$, where p stands for a typical energy/momentum scale. Because we consider the baryons as solitons, we keep only the leading order terms in N_c. In this way, we arrive at the starting Lagrangian.

We quantize the soliton by the collective coordinate quantization, where only the "rotational" modes are treated as dynamical. The resulting Hamiltonian contains a set of new interactions, which have never been considered in the literature. We calculate the matrix elements by using the orthogonality of the irreducible representation of $SU(3)$ and the Clebsch-Gordan coefficients. By using these matrix elements, we calculate the baryon masses in perturbation theory with respect to the symmetry breaking parameter $\delta m \equiv m_s - m$, where m_s is the strange quark mass and m stands for the mass for the up and down quarks. We ignore the isospin breaking in this work.

[a] The χQSM has its own scenario based on chiral symmetry breaking due to instantons. But for our purpose, it is useful to regard it as a Skyrme model.

The calculated masses contain undetermined parameters. In the conventional Skyrme model calculations, they are determined by the profile function of the soliton and the χPT theory parameters. In our effective theory approach, however, they are just parameters to be fitted, because there are infinitely many contributions from higher order terms which we cannot calculate. After fitting the parameters, we make predictions.

2. The Hamiltonian

Let us start with the $SU_f(3)$ χPT action which includes the terms up to $\mathcal{O}(p^4)$[18],

$$S^{\chi\mathrm{PT}} = \frac{F_0^2}{16}\int d^4x \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F_0^2 B_0}{8}\int d^4x \mathrm{Tr}\left(\mathcal{M}^\dagger U + \mathcal{M} U^\dagger\right) + N_c \Gamma[U] + \int d^4x \mathcal{L}_4, \tag{2}$$

where $\mathcal{L}_4 = \sum_{i=1}^{8} L_i \mathcal{O}^i$ is the terms of $\mathcal{O}(p^4)$, $\mathcal{M}$ is the quark mass matrix, $\mathcal{M} = \mathrm{diag}(m, m, m_s)$, and Γ is the WZW term[19,20].

The large-N_c dependence of these low-energy coefficients are known[18,21]:

$$B_0, 2L_1 - L_2, L_4, L_6, L_7 \cdots \mathcal{O}(N_c^0), \tag{3}$$

$$F_0^2, L_2, L_3, L_5, L_8 \cdots \mathcal{O}(N_c^1). \tag{4}$$

As explained in the previous section, we keep only the terms of order N_c. Furthermore, we assume that the constants L_1, L_2 and L_3 have the ratio,

$$L_1 : L_2 : L_3 = 1 : 2 : -6, \tag{5}$$

which is consistent with the experimental values, $L_1 = 0.4 \pm 0.3$, $2L_1 - L_2 = -0.6 \pm 0.5$, and $L_3 = -3.5 \pm 1.1$ (times 10^{-3})[22]. It enables us to write the three terms in a single expression,

$$\sum_{i=1}^{3} L_i \mathcal{O}^i = \frac{1}{32e^2}\mathrm{Tr}\left(\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2\right), \tag{6}$$

where we introduced $L_2 = 1/(16e^2)$. This term is nothing but the Skyrme

term. In this way, we end up with the action,

$$
\begin{aligned}
S[U] = &\frac{F_0^2}{16}\int d^4x \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\int d^4x \mathrm{Tr}\left(\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2\right) \\
&+ N_c\Gamma[U] + \frac{F_0^2 B_0}{8}\int d^4x \mathrm{Tr}\left(\mathcal{M}^\dagger U + \mathcal{M}U^\dagger\right) \\
&+ L_5 B_0 \int d^4x \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right) \\
&+ L_8 B_0^2 \int d^4x \mathrm{Tr}\left(\mathcal{M}^\dagger U \mathcal{M}^\dagger U + \mathcal{M}U^\dagger \mathcal{M}U^\dagger\right), \qquad (7)
\end{aligned}
$$

which is up to including $\mathcal{O}(N_c)$ and $\mathcal{O}(p^4)$ terms. Note that there are tree level contributions to F_π and M_π, and so on. For example,

$$
F_\pi = F_0\left(1 + (2m)L_5\frac{16B_0}{F_0^2}\right). \qquad (8)
$$

This action allows a topological soliton, called “Skyrmion.” The classical hedgehog ansatz,

$$
U_c(\boldsymbol{x}) = \begin{pmatrix} \exp\left(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{x}}F(r)\right) & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0 & 0\end{matrix} & 1 \end{pmatrix}, \qquad (9)
$$

has topological (baryon) number $B = 1$ and stable against fluctuations. We introduce the collective coordinate $A(t)$,

$$
U(t,\boldsymbol{x}) = A(t)U_c(\boldsymbol{x})A^\dagger(t), \qquad (10)
$$

and treat it as a quantum mechanical degree of freedom. By substituting Eq. (10) into Eq. (7), we obtain the following quantum mechanical Lagrangian,

$$
\mathcal{L} = -M_{cl} + \frac{1}{2}\omega^\alpha I_{\alpha\beta}(A)\omega^\beta + \frac{N_c}{2\sqrt{3}}\omega^8 - V(A), \qquad (11)
$$

where ω^α is the “angular velocity,”

$$
A^\dagger(t)\dot{A}(t) = \frac{i}{2}\sum_{\alpha=1}^{8}\lambda_\alpha\omega^\alpha(t). \qquad (12)
$$

In the conventional Skyrme model, all the couplings are given in terms of the χPT parameters and the integrals involving the profile function $F(r)$, which is determined by minimizing the classical energy. In our effective theory approach, on the other hand, they are determined by fitting the physical quantities calculated by using them to the experimental values.

The most important feature of the Lagrangian (11) is that the "inertia tensor" $I_{\alpha\beta}(A)$ depends on A. It has the following form,

$$I_{\alpha\beta}(A) = I^0_{\alpha\beta} + I'_{\alpha\beta}(A), \tag{13}$$

$$I^0_{\alpha\beta} = \begin{cases} I_1\delta_{\alpha\beta} & (\alpha,\beta \in \mathcal{I}) \\ I_2\delta_{\alpha\beta} & (\alpha,\beta \in \mathcal{J}) \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

$$I'_{\alpha\beta}(A) = \begin{cases} \overline{x}\delta_{\alpha\beta}D^{(8)}_{88}(A) & (\alpha,\beta \in \mathcal{I}) \\ \overline{y}d_{\alpha\beta\gamma}D^{(8)}_{8\gamma}(A) & \begin{array}{l}(\alpha \in \mathcal{I},\ \beta \in \mathcal{J} \\ \text{or}\ \ \alpha \in \mathcal{J},\ \beta \in \mathcal{I})\end{array} \\ \overline{z}\delta_{\alpha\beta}D^{(8)}_{88}(A) + \overline{w}d_{\alpha\beta\gamma}D^{(8)}_{8\gamma}(A) & (\alpha,\beta \in \mathcal{J}) \\ 0 & (\alpha = 8 \text{ or } \beta = 8) \end{cases} \tag{15}$$

where $\mathcal{I} = \{1,2,3\}$, $\mathcal{J} = \{4,5,6,7\}$, and $d_{\alpha\beta\gamma}$ is the usual symmetric tensor.

The collective coordinate quantization procedure[23,24,25,26,27] is well-known, and leads to the following Hamiltonian,

$$H = M_{cl} + H_0 + H_1 + H_2, \tag{16}$$

$$H_0 = \frac{1}{2I_1}\sum_{\alpha\in\mathcal{I}}(F_\alpha)^2 + \frac{1}{2I_2}\sum_{\alpha\in\mathcal{J}}(F_\alpha)^2, \tag{17}$$

$$\begin{aligned} H_1 = {} & xD^{(8)}_{88}(A)\sum_{\alpha\in\mathcal{I}}(F_\alpha)^2 + y\left[\sum_{\alpha\in\mathcal{I},\beta\in\mathcal{J}} + \sum_{\alpha\in\mathcal{J},\beta\in\mathcal{I}}\right]\sum_{\gamma=1}^{8} d_{\alpha\beta\gamma}F_\alpha D^{(8)}_{8\gamma}(A)F_\beta \\ & + z\sum_{\alpha\in\mathcal{J}}F_\alpha D^{(8)}_{88}(A)F_\alpha + w\sum_{\alpha,\beta\in\mathcal{J}}\sum_{\gamma=1}^{8} d_{\alpha\beta\gamma}F_\alpha D^{(8)}_{8\gamma}(A)F_\beta \\ & + \frac{\gamma}{2}\left(1 - D^{(8)}_{88}(A)\right), \end{aligned} \tag{18}$$

$$H_2 = v\left(1 - \sum_{\alpha\in\mathcal{I}}\left(D^{(8)}_{8\alpha}(A)\right)^2 - \left(D^{(8)}_{88}(A)\right)^2\right), \tag{19}$$

where

$$x = -\frac{\overline{x}}{2I_1^2}, \quad y = -\frac{\overline{y}}{2I_1I_2}, \quad z = -\frac{\overline{z}}{2I_2^2}, \quad w = -\frac{\overline{w}}{2I_2^2}, \tag{20}$$

and F_α $(\alpha = 1,\cdots,8)$ are the $SU(3)$ generators,

$$[F_\alpha, F_\beta] = i\sum_{\gamma=1}^{8} f_{\alpha\beta\gamma}F_\gamma, \tag{21}$$

where $f_{\alpha\beta\gamma}$ is the totally anti-symmetric structure constant of $SU(3)$. Note that they act on A *from the right*.

3. Fitting the parameters

We calculate the baryon masses (eigenvalues of the Hamiltonian) in perturbation theory. The calculation of the matrix elements of these operators is a hard task and described in Ref. 28 in detail. We consider the mixings of representations among $(\mathbf{8}, \overline{\mathbf{10}}, \mathbf{27})$ for spin-$\frac{1}{2}$ baryons and $(\mathbf{10}, \mathbf{27})$ for spin-$\frac{3}{2}$ baryons.

The best fit set of parameters are obtained by the multidimensional minimization of the evaluation function, $\chi^2 = \sum_i \left(M_i - M_i^{exp}\right)^2 / \sigma_i^2$, where M_i stands for the calculated mass of baryon i, and M_i^{exp}, the corresponding experimental value. How accurately the experimental values should be considered is measured by σ_i. The sum is taken over the octet and decuplet baryons, as well as $\Theta^+(1540)$ and $\phi(1860)$. The results are summarized in the following table.

(MeV)	N	Σ	Ξ	Λ	Δ	Σ^*	Ξ^*	Ω	Θ	ϕ
M_i^{exp}	939	1193	1318	1116	1232	1385	1533	1672	1539	1862
σ_i	0.6	4.0	3.2	0.01	2.0	2.2	1.6	0.3	1.6	2.0
M_i	941	1218	1355	1116	1221	1396	1546	1672	1547	1853

The best fit set of values is

$$\begin{aligned} M_{cl} &= 435\text{MeV},\ I_1^{-1} = 132\text{MeV},\ I_2^{-1} = 408\text{MeV},\ \gamma = 1111\text{MeV}, \\ x &= 14.8\text{MeV},\ y = -33.5\text{MeV},\ z = -292\text{MeV},\ w = 44.3\text{MeV}, \\ v &= -69.8\text{MeV}, \end{aligned} \tag{22}$$

with $\chi^2 = 3.5 \times 10^2$.

Note that they are quite reasonable, though we do not impose any constraint that the higher order (in δm) parameters should be small. The parameter γ is unexpectedly large (even though it is of leading order in N_c), but considerably smaller than the value ($\gamma = 1573$ MeV) for the case (3) of Yabu and Ando. The parameter z seems also too large and we do not know the reason. Our guess is that this is because we do not consider the mixings among an enough number of representations.

4. Predictions and Discussions

We have determined our parameters and now ready to calculate other quantities. First of all, we make a prediction to the masses of the other members

of anti-decuplet,

$$M_{\mathrm{N}'} = 1782 \text{ MeV}, \quad M_{\Sigma'} = 1884 \text{ MeV}. \tag{23}$$

Compare with the chiral quark-soliton model prediction[29],

$$M_{\mathrm{N}'} = 1646 \text{ MeV}, \quad M_{\Sigma'} = 1754 \text{ MeV}. \tag{24}$$

It is interesting to note that Σ' is heavier than ϕ.

The decay widths are such quantities that can be calculated. The results are reported in Ref. 28.

What should we do to improve the results? First of all, we should include more (arbitrarily many(?)) representations. The mixings with other representations are quite large, so that we expect large mixings with the representations we did not include. Second, we may have a better fitting procedure. In the present method, all of the couplings are treated equally. The orders of the couplings are not respected. Third, in order to understand the narrow width of Θ^+, we might have to consider general N_c multiplets[30]. Finally it seems interesting to include "radial" modes[31].

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